Fuzzy UML

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Übersicht

- Introduction
- Basic knowledge
- Fuzzy objects und classes
- UML Modeling of Fuzzy Data
  - Fuzzy Class
  - Fuzzy Generalization
  - Fuzzy Aggregation
  - Fuzzy Association
  - Fuzzy Dependency
Introduction

Imperfection?
- Imprecision, vagueness
  - \( x \in (10, 15) \)......

Uncertainty
- \( \text{Das Sparschwein erhält mehr als 50 Euro.} \)

Fuzzy?
- \( \text{Grosse Person, schnelle Autos} \).....

Information in real-world

- Requirements
  - Collection & Analysis
    - Conceptual Data Modeling
      - Relational DB
        - Logical Database Model
          - Physical Database Model

Database Design Process
Basic knowledge

- UML Class Model
  - Class

<table>
<thead>
<tr>
<th>Class name</th>
<th>Attributes</th>
<th>Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interior</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chassis</td>
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</tbody>
</table>

- Relationships
  1. Aggregation
Basic knowledge

2. Generalization

Vehicle

Car
Truck

3. Association

CD Player installing Car

4. Dependency

Dependent Employee
Basic knowledge

- Fuzzy Set and Possibility Distribution

Crisp Set to Concept "Grosse Person"

Fuzzy Set to Concept "Grosse Person"
Basic knowledge

\( F : \text{Fuzzy Set} \)

\( U : \text{a universe of discourse} \quad \text{Membership function } \mu_F : U \rightarrow [0, 1] \)

\( \mu_F(u), \text{for each } u \in U : \text{the degree of membership of } u \text{ in the fuzzy set } F. \)

\[ F = \{ \mu(u_1)/u_1, \mu(u_2)/u_2, \ldots, \mu(u_n)/u_n \} \]

\( \prod_x : \text{Possibility Distribution} \)

\( X : \text{Variable} \)

\( \prod_x(u_i), u_i \in U : \text{the possibility that } u_i \text{ is true.} \)

\[ \prod_x = \{ \prod_x(u_1)/u_1, \prod_x(u_2)/u_2, \ldots, \prod_x(u_n)/u_n \} \]

\( \prod_x = F \quad \text{is true} \)
Basic knowledge

- Semantic measure of fuzzy data
  
  \[ \text{SID} : \text{semantic inclusion degree} \]
  
  \[ U = \{u_1, u_2, \ldots, u_n\} \]
  
  \[ \prod_A, \prod_B : \text{fuzzy data on } U \quad \prod_A(u_i), u_i \in U \]
  
  \[ \text{SID} \left( \prod_A, \prod_B \right) = \frac{\sum_{i=1}^{n} \min_{u_i, u_j \in D} (\prod_B(u_i), \prod_A(u_i))}{\sum_{i=1}^{n} \prod_B(u_i)}. \]
  
  \[ \text{SID}([0.6/27, 0.8/28, 0.9/29, 1.0/30], [0.6/25, 0.8/26, 1.0/27, 0.9/28]) = (0.6+0.8)/(0.6+0.8+1.0+0.9) = 0.42 \]
Fuzzy objects und classes

- Fuzzy objects
  - lack of information
  - at least one attribute whose value is a fuzzy set

\[ O_1(\text{Age}) = \{ 0.6/25, 0.8/26, 0.9/28 \} \]
Fuzzy objects und classes

- Fuzzy classes
  - 2 different view points
    1. an extensional class
       \[ C = \{ O_1, O_2, \ldots, O_m \} \]
    2. an intensional class
       \[ C = \{ A_1, A_2, \ldots, A_n \} \]

special case:
Fuzzy objects und classes

- A class is fuzzy because of:
  1. defined by some fuzzy object
     \[
     C = \{ O_1, O_2, \ldots, O_m \}
     \]
  2. the domain of an attribute may be fuzzy
     \[
     C = \{ A_1, A_2, \ldots, A_n \}
     \]
     \[
     \text{cdom}(\text{Age}) = \{ 5 - 20 \}
     \]
     \[
     \text{fdom}(\text{Age}) = \{ \{1.0/20, 1.0/21, 0.7/22\},
     \{0.4/22, 0.6/23, 0.8/24, 1.0/25\},
     \{0.6/27, 0.8/29, 1.0/30\} \}
     \]
Fuzzy objects und classes

3.

Specialization

Generalization

Super

Sub

Sub

Super

Sub

Sub
Fuzzy objects und classes

Fuzzy object – class relationships

- **a)** Crisp class and crisp object
- **b)** Crisp class and fuzzy object:  
  - *fuzzy relationship*
- **c)** Fuzzy class and crisp object:  
  - *fuzzy relationship*
- **d)** Fuzzy class and fuzzy object:  
  - *fuzzy relationship*

  *fuzzy relationship* → the object belongs to the class with \( \mu \in [0, 1] \)

- \( \mu_C(o) = ? \)

  \(
  C: \quad \text{a class with attributes}\{A_1, A_2, ..., A_n\}
  \)

  \(
  o(A_i): \quad \text{the attribute value of } o \text{ on } A_i
  \)
Fuzzy objects und classes

\( \mu_c(o) = ? \)

\[
\mu_c(o) = \frac{\sum_{i=1}^{n} \text{ID}(\text{dom}(A_i), o(A_i)) \cdot w(A_i(C))}{\sum_{i=1}^{n} w(A_i(C))}
\]

- \( w(A_i(C)) \): the weight of attribute \( A_i \) to class \( C \)
- \( \text{dom}(A_i) = \text{cdom}(A_i) \cup \text{fdom}(A_i) \)
- \( \text{cdom}(A_i) = \{ c_1, c_2, ..., c_k \} \quad \text{fdom}(A_i) = \{ f_1, f_2, ..., f_m \} \)

\[
\text{ID}(\text{dom}(A_i), o(A_i)) = \max(\text{ID}(\text{cdom}(A_i), o(A_i)), \text{ID}(\text{fdom}(A_i), o(A_i)))
\]

\[
= \max(\text{SID}([1.0/c_1, 1.0/c_2, ..., 1.0/c_k], o(A_i)), \max_j(\text{SID}(f_j, o(A_i))))
\]
UML Modeling of Fuzzy Data

- The three levels of fuzziness
  1. The class belongs to the data model? or The attribute belongs to the class?
     \[ \text{WITH mem DEGREE} \]
  2. Are the instances instances of the class?
     \[ \mu \]
  3. Are attribute values of the instances of the class fuzzy?
     \[ A \text{ keyword FUZZY} \]
UML Modeling of Fuzzy Data

- Fuzzy Class

<table>
<thead>
<tr>
<th>ID</th>
<th>Name</th>
<th>Fuzzy Age</th>
<th>Office</th>
<th>μ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>WITH 0.8 DEGREE</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
UML Modeling of Fuzzy Data

Fuzzy Generalization

- Based on the extensional viewpoint of class
  - With the second level of fuzziness:
    \[ (\forall e) \ (\beta \leq \mu_B(e) \leq \mu_A(e)) \]
    \[ \beta: \text{a given threshold} \]
    The membership degree that \( B \) is a subclass of \( A \) should be:
    \[ \min_{\mu_B(e) \geq \beta} (\mu_B(e)) \]
  - With the first level of fuzziness:
    \[ (\forall e) \ (\beta \leq \mu_B(e) \leq \mu_A(e)) \] \[ (\beta \leq \text{degree}_B \leq \text{degree}_A) \]
    \[ (\forall e) \ (\max (\mu_B_1(e), \mu_B_2(e), ..., \mu_B_n(e)) \leq \mu_A(e)) \] \[ (\max \ (\text{degree}_B_1, \text{degree}_B_2, ..., \text{degree}_B_n) \leq \text{degree}_A) \]
UML Modeling of Fuzzy Data

- Based on the intensional viewpoint of class
  - With the second level of fuzziness:
    \[ \mu(A, B) \geq \beta \]
  - With the first level of fuzziness:
    \[ (\mu(A, B) \geq \beta) \land (\beta \leq \text{degree}_B \leq \text{degree}_A) \]

![Diagram showing UML modeling of fuzzy data categories: Youth, Young Student, Young Faculty]
UML Modeling of Fuzzy Data

Fuzzy Aggregation

A: an aggregation of constituent parts B1, B2, ..., and Bn.
For e ∈ A, the projection of e to Bi is denoted by (e ↓ Bi) ∈ Bi.

- Based on the extensional viewpoint of class
  - With the second level of fuzziness:

\[(\forall e) (e \in A \quad \beta \leq \mu_A(e) \leq \min (\mu_{B1}(e \downarrow B1), \mu_{B2}(e \downarrow B2), \ldots, \mu_{Bn}(e \downarrow Bn)))\]

the membership degree that A is an aggregation of class sets B1, B2, ..., and Bn:

\[\min_{1 \leq i \leq n} \mu_{Bi}(e \downarrow Bi) \geq \beta (\mu_{Bi}(e \downarrow Bi))\]
UML Modeling of Fuzzy Data

- With the first level of fuzziness:

\[(\forall e) \ (e \in A) \quad \beta \leq \mu_A (e) \leq \min (\mu_{B1} (e \downarrow B1), \mu_{B2} (e \downarrow B2), ..., \mu_{Bn} (e \downarrow Bn)) \quad \text{degree}_A \leq \min (\text{degree}_B1, \text{degree}_B2, ..., \text{degree}_Bn)\]

- Based on the intensional viewpoint of class

\[A \downarrow Bi : \text{the projection of } A \text{ to } Bi\]

\[\mu(Bi, A \downarrow Bi) \ (1 \leq I \leq n): \text{the degree to which } Bi \text{ semantically includes } A \downarrow Bi\]

- With the second level of fuzziness:

\[\min (\mu (B1, A \downarrow B1), \mu (B2, A \downarrow B2), ..., \mu (Bn, A \downarrow Bn)) \geq \beta\]
UML Modeling of Fuzzy Data

- With the first level of fuzziness:

\[
\min (\mu (B_1, A \downarrow_{B_1}), \mu (B_2, A \downarrow_{B_2}), \ldots, \mu (B_n, A \downarrow_{B_n})) \geq \beta \land \\
\text{degree}_A \leq \min(\text{degree}_{B_1}, \text{degree}_{B_2}, \ldots, \text{degree}_{B_n})
\]
UML Modeling of Fuzzy Data

**Fuzzy Association**

The first level of fuzziness:

(a)

The second level of fuzziness:

(b)

(a) + (b):

(c)
UML Modeling of Fuzzy Data

$ass(A, B)$: the association relationship between $A$ and $B$

$ass(e, f)$: the association relationship between $e$ and $f$

Classes with 2. level of fuzziness → the 2. level of fuzziness in the association

$$\mu(ass(e,f)) = \min(\mu_A(e), \mu_B(f))$$

Classes with 1. level of fuzziness → the 1. level of fuzziness in the association

$$\mu(ass(e,f)) = degree_{ass}$$

$$degree_{ass} = \min(degree_A, degree_B)$$

Classes with 1. level and the 2. level of fuzziness:

$$\mu(ass(e,f)) = \min(\mu_A(e), \mu_B(f), degree_A, degree_B, degree_a)$$
UML Modeling of Fuzzy Data

Fuzzy Dependency

A fuzzy UML data Model
UML Modeling of Fuzzy Data

The definition of a fuzzy class:

CLASS Young Students WITH DEGREE OF 1.0
INHERITS Students WITH DEGREE OF 1.0
ATTRIBUTES
  ID: TYPE OF string WITH DEGREE OF 1.0
  Name: TYPE OF string WITH DEGREE OF 1.0
  Age: FUZZY DOMAIN {very Young, zoug, old, very old}: TYPE OF integer WITH DEGREE OF 1.0
  Height: DOMAIN [60, 210]: TYPE OF real WITH DEGREE OF 1.0
    Membership_Attribute name
  WEIGHT
    w(ID) = 0.1
    w(Name) = 0.1
    w(Age) = 0.9
    w(Height) = 0.2
METHODS
...
END