Hauptseminar

Imperfektion und erweiterte Konzepte im Data Warehousing

Fuzzy UML

Seminararbeit

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Wang Lu

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Fuzzy UML

The UML (Unified Modeling Language) is a set of object-oriented modeling notations and is a standard of the Object Data Management Group (ODMG). It can be applied in many areas of software engineering and knowledge engineering. Increasingly, the UML is being applied to data modeling. However, information in real-world applications is often vague or ambiguous. In this chapter, different levels of fuzziness are introduced into the class of the UML and the corresponding graphical representations are given. The class diagrams of the UML can hereby model fuzzy information.

6.1 Introduction

The objective of database design is to capture the essential aspects of some real-world enterprise for which one wishes to construct a database. Figure 6.1 shows a simplified description of the database design process. Then four major steps are applied for the database design process, which are the requirements collection & analysis, conceptual data modeling, logical database model, and physical database model, respectively [Ma05a].

In the first step, the database designers collect and analyze the data requirements from prospective database users.
In the second step, the conceptual data models (e.g., UML) are used to create a conceptual schema for the database.
In the third step, the logical database model is designed through mapping the conceptual schema represented by the conceptual data model. The result of this step is perhaps a relational or object-oriented database model.
Finally, in the fourth step, the physical database model is design.
In real-world applications, information is often imperfect. There have been some attempts at classifying various possible kinds of imperfect information. Inconsistency, imprecision, vagueness uncertainty, and ambiguity are five basic kinds of imperfect information in database systems.

Inconsistency is a kind of semantic conflict when one aspect of the real world is irreconcilably represented more than once in a database or several different databases. For example, one has "married" value and "single" value for Tom's marital status.

Intuitively, imprecision and vagueness are relevant to the content of an attribute value, and it means that a choice must be made from a given range of values but it is not known exactly which one to choose at present. For example, "between 20 and 30 years old" and "young" for the attribute Age are imprecise and vague values, respectively.

Uncertainty is related to the degree of truth of its attribute value, and it means that one can apportion some, but not all, of our belief to a given value or a group of values. For example, the sentence that "I am 95 percent sure that Tom is married" represents information uncertainty.

The ambiguity means that some elements of the model lack complete semantics, leading to several possible interpretations.

Vagueness and uncertainty are generally modeled with fuzzy sets and possibility theory. Many of the existing approaches dealing with imprecision and uncertainty are based on the theory of fuzzy sets.

The remainder of this chapter is organized as follows. The second section gives the basic knowledge about fuzzy data and semantic measure as well as knowledge of
6.2 Basic knowledge

6.2.1 Fuzzy Set and Possibility Distribution

The concept of fuzzy sets was originally introduced by Zadeh. Let $U$ be a universe of discourse. A fuzzy value on $U$ can be characterised by a fuzzy set $F$ in $U$. A membership function $\mu_F : U \rightarrow [0,1]$ is defined for the fuzzy set $F$, where $\mu_F(u)$, for each $u \in U$, denotes the degree of membership of $u$ in the fuzzy set $F$. Thus, the fuzzy set $F$ is described as follows [Ma05b]:

$$ F = \{ \mu(u_1)/u_1, \mu(u_2)/u_2, \ldots, \mu(u_n)/u_n \} $$

where the pair $\mu(u_i)/u_i$ represents the value $u_i$ and its membership degree $\mu(u_i)$. The membership function $\mu_F(u)$ can be interpreted as a measure of the possibility that the value of variable $X$ is $u$. A fuzzy set is equivalently represented by its associated possibility distribution $\pi_X$:

$$ \pi = \{ \pi_X(u_1)/u_1, \pi_X(u_2)/u_2, \ldots, \pi_X(u_n)/u_n \} $$

Here, $\pi_X(u_i), u_i \in U$, denotes the possibility that $u_i$ is true. Let $\pi_X$ and $F$ be the possibility distribution representation and the fuzzy set representation for a fuzzy value, respectively. It is apparent that $\pi_X = F$ is true.

Beispiel 6.2.1 In the case of the "tall person" concept a vague concept. This concept can be elegantly represented by means of a Fuzzy set, as Figure 6.2 shows.
Semantic measure of fuzzy data [MZM04] Let $U = \{u_i, u_2, \ldots, u_n\}$ be the universe of discourse. Let $\pi_A$ and $\pi_B$ be two fuzzy data on $U$ based on possibility distribution and $\pi_A(u_i)$, $u_i \in U$, denote the possibility that $u_i$ is true. Let $Res$ be a resemblance relation on domain $U$, $\alpha$ for $0 \leq \alpha \leq 1$ be a threshold corresponding to $Res$. $\text{SID}(\pi_A, \pi_B)$ is then defined by

$$\text{SID}(\pi_A, \pi_B) = \frac{\sum_{i=1}^{n} \min_{u_i, u_j \in D} (\pi_B(u_i), \pi_A(u_i))}{\sum_{i=1}^{n} \pi_B(u_i)}.$$ 

The notion of the semantic equivalence degree of attribute values can be extended to the semantic equivalence degree of tuples. Let $t_i = \langle a_{i1}, a_{i2}, \ldots, a_{in} \rangle$ and $t_j = \langle a_{j1}, a_{j2}, \ldots, a_{jn} \rangle$ be two tuples in fuzzy relational instance $r$ over schema $R(A_1, A_2, \ldots, A_n)$. The semantic equivalence degree of tuples $t_i$ and $t_j$ is denoted $\text{SE}(t_i, t_j) = \min\{\text{SE}(t_i[A_1], t_j[A_1]), \text{SE}(t_i[A_2], t_j[A_2]), \ldots, \text{SE}(t_i[A_n], t_j[A_n])\}$.

6.2.2 UML Class Model

UML provides a collection of models to capture the many aspects of a software system. From the database modeling point of view, the most relevant model is the class model. The building blocks in this class model are those of classes and relationships. We briefly review these building blocks.

**Classes** Being the descriptor for a set of objects with similar structure, behavior, and relationships, a class represents a concept within the system being modeled. Classes have data structure and behavior and relationships to other elements. Figure 6.3 shows a class.

**Relationships** Another main structural component in the class diagram of the UML is relationships for the representation of relationship between classes or class instances. UML supports a variety of relationships:

1. *Aggregation and composition:* An aggregation captures a whole-part relationship between an aggregate, a class that represent the whole, and a constituent
2. **Generalization:** Generalization is used to define a relationship between classes to build taxonomy of classes: one class is a more general description of a set of other classes. Figure 6.5 shows a generalization relationship.

3. **Association:** Associations are relationships that describe connections among class instances. A role may be assigned to each class taking part in an association, making the association a directed link. Figure 6.6 shows an association relationship.

4. **Dependency:** A dependency indicates a semantic relationship between two classes. It indicates a situation in which a change to the target class may require a change to the source class in the dependency. Figure 6.7 shows a dependency relationship.
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Abbildung 6.6: Simple association relationship

Abbildung 6.7: Simple dependency relationship

6.3 Fuzzy objects and classes

6.3.1 Fuzzy objects

An object is fuzzy because of a lack of information. For example, an object representing a part in preliminary design for a certain will also be made of stainless steel, moulded steel, or alloy steel (each of them may be connected with a possibility, say, 0.7, 0.5 and 0.9, respectively). Formally, objects that have at least one attribute whose value is a fuzzy set are fuzzy objects.

6.3.2 Fuzzy classes

Theoretically, a class can be considered from two different viewpoints:

1. an extensional class, where the class is defined by the list of its object instances, and

2. an intensional class, where the class is defined by a set of attributes and their admissible values.

In addition, a subclass defined from its superclass by means of inheritance mechanism in the OODB can be seen as the special case of (b) above.

Therefore, a class is fuzzy because of the following several reasons. First, some objects are fuzzy ones, which have similar properties. A class defined by these
objects may be fuzzy. These objects belong to the class with membership degree of \([0, 1]\). Second, when a class is intensionally defined, the domain of an attribute may be fuzzy and a fuzzy class is formed. Third, the subclass produced by a fuzzy class by means of specialization and the superclass produced by some classes (in which there is at least one class who is fuzzy) by means of generalization are also fuzzy.

6.3.3 Fuzzy object-class relationships

In the fuzzy OODB, the following four situations can be distinguished for object-class relationships.

1. *Crisp class and crisp object*: the object belongs or not to the class certainly.

2. *Crisp class and fuzzy object*: the object may be related to the class with the special degree in \([0, 1]\).

3. *Fuzzy class and crisp object*: the object may belong to the class with the membership degree in \([0, 1]\).

4. *Fuzzy class and fuzzy object*: the object belongs to the class with the membership degree in \([0, 1]\).

The object-class relationships in (b)–(d) above are called *fuzzy object-class relationships*. In fact, the situation in (a) can be seen as the special case of fuzzy object-class relationships, where the membership degree of the object to the class is one.

In order to calculate the membership degree of an object to the class in a fuzzy object-class relationship, it is necessary to evaluate the degrees that the attribute domains of the class include the attribute values of the object. The attributes play different role in the definition and identification of a class, therefore, a weight \(w\) is assigned to each attribute of the class according to its importance by designer. Then the membership degree of an object to the class in a fuzzy object-class relationship should be calculated using the inclusion degree of object values with respect to the class domains and the weight of attributes.

Let \(C\) be a class with attributes \(\{A_1, A_2, \ldots, A_n\}\), \(o\) be an object on attribute set \(\{A_1, A_2, \ldots, A_n\}\), and let \(o(A_i)\) denote the attribute value of \(o\) on \(A_i (1 \leq i \leq n)\). In \(C\), each attribute \(A_i\) is denoted \(\text{ID}(\text{dom}(A_i), o(A_i))\). In the following, we investigate the evaluation of \(\text{ID}(\text{dom}(A_i), o(A_i))\).

Let \(\text{fdom}(A_i) = \{f_1, f_2, \ldots, f_m\}\), where \(f_j (1 \leq j \leq m)\) is a fuzzy value, and \(\text{cdom}(A_i) = \{c_1, c_2, \ldots, c_k\}\), where \(c_l (1 \leq l \leq k)\) is a crisp value. Then

\[
\text{ID}(\text{dom}(A_i), o(A_i)) = \max(\text{ID}(\text{cdom}(A_i), o(A_i)), \text{ID}(\text{fdom}(A_i), o(A_i)))
\]

\[
= \max(\text{SID}(\{1.0/c_1, 1.0/c_2, \ldots, 1.0/c_k\}, o(A_i)),
\max_j(\text{SID}(f_j, o(A_i))))
\]
Now, we define the formula to calculate the membership degree of the object \( o \) to the class \( C \) as follows, where \( w(A_i(C)) \) denotes the weight of attribute \( A_i \) to class \( C \):

\[
\mu_C(o) = \frac{\sum_{i=1}^{n} \text{ID}(\text{dom}(A_i), o(A_i)) \times w(A_i(C))}{\sum_{i=1}^{n} w(A_i(C))}
\] (6.1)

### 6.4 Fuzzy Modeling of Fuzzy Data

We define three levels of fuzziness:

1. Fuzziness in the extent to which the class belongs in the data model as well as fuzziness on the content (in terms of attributes) of the class

2. Fuzziness related to whether some instances are instances of a class

3. The third level of fuzziness is on attribute values of the instances of the class; an attribute in a class defines a value domain, and when this domain is a fuzzy subset or a set of fuzzy subset, the fuzziness of an attribute value appears

#### 6.4.1 Fuzzy Class

In order to model the first level of fuzziness, i.e., an attribute or a class with degree of membership, the attribute or class name should be followed by a pair of words WITH mem DEGREE, where \( 0 \leq \text{mem} \leq 1 \). In order to model the third level of fuzziness, a keyword FUZZY is placed in front of the attribute. In the second level of fuzziness, an additional attribute \((\mu)\) is introduced into the class to represent instance membership degree to the class, with an attribute domain that is \([0, 1]\). In order to differentiate the class with the second level of fuzziness, we use a dashed-outline rectangle to denote such class. Figure 6.8 shows a fuzzy class Ph.D.student.
6.4.2 Fuzzy Generalization

A new class, called subclass, is produced form another class, called superclass.

- **Based on the extensional viewpoint of class:**
  We can assess fuzzy subclass-superclass relationship by utilizing the inclusion degree of objects to the class.

Because a subclass is the specialization of the superclass, any one object belonging to the subclass must belong to the superclass. This characteristic can be used to determine if two classes have a subclass-superclass relationship. A subclass produced from a fuzzy superclass must be fuzzy, the subclass-superclass relationship is fuzzy too. In other words, a class is a subclass of another class with membership degree of $[0, 1]$ at this moment. Correspondingly, we have the following method for determining a subclass-superclass relationship:

- We assume that the classes can only have the second level of fuzziness.
  1. For an (fuzzy) object, if the membership degree that it belongs to the subclass is less than or equal to the membership degree, then it belongs to the superclass.
  2. The membership degree that it belongs to the subclass is greater than or equal to the given threshold.

Formally, let $A$ and $B$ be (fuzzy) classes and $\beta$ be a given threshold. We say $B$ is a subclass of $A$ if

$$\forall e (\beta \leq \mu_B(e) \leq \mu_A(e))$$

The membership degree that is a $B$ subclass of $A$ should be $\min_{\mu_B(e) \geq \beta}(\mu_B(e))$. Here, $e$ is the object instance of $A$ and $B$ in the universe of discourse, and $\mu_A(e)$ and $\mu_B(e)$ are membership degrees of $e$ to $A$ and $B$, respectively.

- We consider the situation that classes $A$ or $B$ are the classes with membership degree, namely, with the first level of fuzziness.

$$A \text{ WITH } degree_A \text{ DEGREE}$$
$$B \text{ WITH } degree_B \text{ DEGREE}$$

Then $B$ is a subclass of $A$ if

$$\forall e (\beta \leq \mu_B(e) \leq \mu_A(e)) \land (\beta \leq degree_B \leq degree_A)$$

The membership degrees of $A$ and $B$ must be greater than or equal to the given threshold, and the membership degree of $A$ must be greater than or equal to the membership degree of $B$. 
Consider a fuzzy superclass $A$ and its fuzzy subclasses $B_1, B_2, \ldots, B_n$ with instance membership degrees $\mu_A, \mu_{B_1}, \mu_{B_2}, \ldots, \mu_{B_n}$, respectively, which may have the degrees of membership $\text{degree}_A, \text{degree}_{B_1}, \text{degree}_{B_2}, \ldots,$ and $\text{degree}_{B_n}$, respectively. Then the following relationship is true:

$$(\forall e)(\max(\mu_{B_1}(e), \mu_{B_2}(e), \ldots, \mu_{B_n}(e)) \leq \mu_A(e)) \land (\max(\text{degree}_{B_1}, \text{degree}_{B_2}, \ldots, \text{degree}_{B_n}) \leq \text{degree}_A)$$

- **Based on the intensional viewpoint of class:**
  We can use the inclusion degree of a class with respect to another class to determine the relationships between fuzzy subclass and superclass.

  - We assume that classes $A$ and $B$ can only have the second level of fuzziness.
    Formally, let $A$ and $B$ be (fuzzy) classes and the degree that $B$ is the subclass of $A$ be denoted by $\mu(A, B)$. For a given threshold $\beta$, we say $B$ is a subclass of $A$ if
      $$\mu(A, B) \geq \beta$$
    - We consider the situation that classes $A$ or $B$ are the classes with membership degree, namely, with the first level of fuzziness:
      $$A \text{ WITH degree}_A \text{ DEGREE}$$
      $$B \text{ WITH degree}_B \text{ DEGREE}$$
    Then $B$ is a subclass of $A$ if
      $$(\mu(A, B) \geq \beta) \land (\beta \leq \text{degree}_B \leq \text{degree}_A)$$

The inclusion degree of a (fuzzy) subclass with respect to the (fuzzy) superclass can be calculated according to the inclusion degree of the attribute domains of the subclass with respect to the attribute domains of the superclass as well as the weight of attributes(s.formel 6.1 on page 8).

In order to represent a fuzzy generalization relation, a dashed peculiar triangular arrowhead is applied. Figure 6.9 shows a fuzzy generalization relationship. Classes young Student and Young Faculty are all classes with the second level of fuzziness. These classes may have some instances that belong to the classes with membership degree. These two classes can be generalized into class Youth, a class with the second level of fuzziness.

### 6.4.3 Fuzzy Aggregation

An aggregation captures a whole-part relationship between an aggregate and a constituent part.
Based on the extensional viewpoint of class: Every instance of an aggregate can be projected into a set of instances of constituent parts. Let $A$ be an aggregation of constituent parts $B_1, B_2, \ldots, B_n$. For $e \in A$, the projection of $e$ to $B_i$ is denoted by $e \downarrow B_i$. Then we have $(e \downarrow B_1) \in B_1, (e \downarrow B_2) \in B_2, \ldots, (e \downarrow B_n) \in B_n$.

A class aggregated from fuzzy constituent parts must be fuzzy, the aggregation is fuzzy too. At this point, a class is an aggregation of constituent parts with membership degree of $[0, 1]$. Correspondingly, we have the following method for determining a fuzzy aggregation relationship:

1. For any (fuzzy) object, if the membership degree to which it belongs to the aggregate is less than or equal to the membership degree to which its projection to each constituent part belongs to the corresponding constituent part.

2. The membership degree to which it belongs to the aggregate is greater than or equal to the given threshold.

The aggregate is then an aggregation of the constituent parts with the membership degree, which is the minimum in the membership degrees to which the projections of these objects to these constituent parts belong to the corresponding constituent parts.

Let $A$ be a fuzzy aggregation of fuzzy class sets $B_1, B_2, \ldots, B_n$, with instance membership degrees that are $\mu_A, \mu_{B_1}, \mu_{B_2}, \ldots, \mu_{B_n}$, respectively. Let $\beta$ be a given threshold. Then,

$$(\forall e)(e \in A \land \beta \leq \mu_A(e) \leq \min(\mu_{B_1}(e \downarrow B_1), \mu_{B_2}(e \downarrow B_2), \ldots, \mu_{B_n}(e \downarrow B_n)))$$

The membership degree that $A$ is an aggregation of class sets $B_1, B_2, \ldots, B_n$ should be $\min_{\mu_{B_i}(e \downarrow B_i) \geq \beta}(\mu_{B_i}(e \downarrow B_i))(1 \leq i \leq n)$. Here, $e$ is object instance of $A$.

We consider the first level of fuzziness in the classes $A, B_1, B_2, \ldots, and
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$B_n$, namely, they are the fuzzy classes with membership degrees.

\[
A \text{ WITH degree}_A \text{ DEGREE},
\]

\[
B_1 \text{ WITH degree}_{B_1} \text{ DEGREE},
\]

\[
B_2 \text{ WITH degree}_{B_2} \text{ DEGREE},
\]

\[
\ldots .
\]

\[
B_n \text{ WITH degree}_{B_n} \text{ DEGREE}.
\]

Then $A$ is an aggregate of $B_1, B_2, \ldots, B_n$ if

\[
(\forall e \in A \land \beta \leq \mu_A(e) \leq \min(\mu_{B_1}(e \downarrow_{B_1}), \mu_{B_2}(e \downarrow_{B_2}), \ldots, \mu_{B_n}(e \downarrow_{B_n})) \land
degree_A \leq \min(\text{degree}_{B_1}, \text{degree}_{B_2}, \ldots, \text{degree}_{B_n})).
\]

- **Based on the intensional viewpoint of class:**

  Let $A$ be a fuzzy aggregation of fuzzy class sets $B_1, B_2, \ldots, B_n$, and $\beta$ be a given threshold. Also let the projection of $A$ to $B_i$ be denoted by $A \downarrow_{B_i}$.

  Then,

  - We assume that the classes $A, B_1, B_2, \ldots, B_n$ can only have the second level of fuzziness.

    \[
    \min(\mu(B_1, A \downarrow_{B_1}), \mu(B_2, A \downarrow_{B_2}), \ldots, \mu(B_n, A \downarrow_{B_n})) \geq \beta
    \]

    Here $\mu(B_i, A \downarrow_{B_i})(1 \leq i \leq n)$ is the degree to which $B_i$ semantically includes $A \downarrow_{B_i}$. The membership degree to which $A$ is an aggregation of $B_1, B_2, \ldots, B_n$ is $\min(\mu(B_1, A \downarrow_{B_1}), \mu(B_2, A \downarrow_{B_2}), \ldots, \mu(B_n, A \downarrow_{B_n}))$.

  - We consider the first level of fuzziness in the classes $A, B_1, B_2, \ldots, B_n$, namely, they are the fuzzy classes with membership degrees.

    \[
    A \text{ WITH degree}_A \text{ DEGREE},
    \]

    \[
    B_1 \text{ WITH degree}_{B_1} \text{ DEGREE},
    \]

    \[
    B_2 \text{ WITH degree}_{B_2} \text{ DEGREE},
    \]

    \[
    \ldots .
    \]

    \[
    B_n \text{ WITH degree}_{B_n} \text{ DEGREE}.
    \]

    Then $A$ is an aggregate of $B_1, B_2, \ldots, B_n$ if

    \[
    \min(\mu(B_1, A \downarrow_{B_1}), \mu(B_2, A \downarrow_{B_2}), \ldots, \mu(B_n, A \downarrow_{B_n})) \geq \beta \land
    degree_A \leq \min(\text{degree}_{B_1}, \text{degree}_{B_2}, \ldots, \text{degree}_{B_n})
    \]

  A dashed open diamond is used to denote a fuzzy aggregate relationship. A fuzzy aggregation relationship is shown in Figure 6.10. A car is aggregated by engine, interior and chassis. The engine is old, and we have a fuzzy class *Old Engine* with the second level of fuzziness. Class *Old Car* aggregated by classes *interior* and *chassis* and fuzzy class *old engine* is a fuzzy one with the second level of fuzziness.
6.4 Fuzzy Modeling of Fuzzy Data

6.4.4 Fuzzy Association

Two levels of fuzziness can be identified in the association relationship. The first level of fuzziness means that an association relationship fuzzily exists in two associated classes, namely, this association relationship occurs with a degree of possibility. Also, it is possible that it is unknown for certain if two class instances respectively belonging to the associated classes have the given association relationship, although this association relationship must occur in these two classes. This is the second level of fuzziness in the association relationship and is caused because an instance belongs to a given class with membership degree. It is possible that the two levels of fuzziness mentioned above may occur in an association relationship simultaneously.

We can place a pair of words WITH mem DEGREE \((0 \leq mem \leq 1)\) after the role name of an association relationship to represent the first level of fuzziness in the association relationship. We use a double line with an arrowhead to denote the second level of fuzziness in the association relationship. Figure 6.11 shows two levels of fuzziness in fuzzy association relationships. In part (a), it is uncertain if the CD player is installed in the car, and the possibility is 0.8. In part (b), it is certain that the CD player is installed in the car, and the possibility is 1.0. But at the level of instances, there exists the possibility that the instances of classes CD Player and Car may or may not have the association relationship installing. In part (c), two kinds of fuzzy association relationships in parts (a) and (b) arise simultaneously.

It has been shown above that three levels of fuzziness can occur in classes.

- Classes with the second level of fuzziness generally result in the second level of fuzziness in the association, if this association definitely exists.

Let \(A\) and \(B\) be two classes with the second level of fuzziness. Then, the instance \(e\) of \(A\) is one with membership degrees \(\mu_A(e)\), and the instance \(f\)
of $B$ is one with membership degrees $\mu_B(f)$. Assume that the association relationship between $A$ and $B$, denoted $ass(A, B)$, is one without the first level of fuzziness. It is clear that the association relationship between $e$ and $f$, denoted $ass(e, f)$, is one with the second level of fuzziness, the membership degree can be calculated by the following:

$$\mu(ass(e, f)) = \min(\mu_A(e), \mu_B(f))$$

- The classes with the first level of fuzziness generally result in the first level of fuzziness of the association, if this association is not indicated explicitly. Let $A$ and $B$ be two classes only with the first level of fuzziness, denoted $A$ WITH $degree\_A$ DEGREE and $B$ WITH $degree\_B$ DEGREE, respectively. Then the association relationship between $A$ and $B$, denoted $ass(A, B)$, is one with the first level of fuzziness, namely, $ass(A, B)$ WITH $degree\_ass$ DEGREE. For the instance $e$ of $A$ and the instance $f$ of $B$, in which $\mu_A(e) = 1.0$ and $\mu_B(f) = 1.0$, we have:

$$\mu(ass(e, f)) = degree\_ass = \min(\mu_A, \mu_B)$$

- Let us focus on a situation in which the classes are the first level and the second level of fuzziness. Let $A$ and $B$ be two classes with the first level of fuzziness, denoted $A$ WITH $degree\_A$ DEGREE and $B$ WITH $degree\_B$ DEGREE, respectively. Let $ass(A, B)$ be the association relationship with the first level of fuzziness.
6.4.5 Fuzzy Dependency

The dependency relationship is only related to the classes and does not require a set of instances for its meaning. Therefore, the second-level fuzziness and the third-level fuzziness in class do not affect the dependency relationship.

Fuzzy dependency relationship is a dependency relationship with a degree of possibility. Assume that the source class is fuzzy, with the first level of fuzziness. The target class must be fuzzy, with the first level of fuzziness. The degrees of possibility that the target class is decided by the source class are the same as the membership degrees of source classes. For source class Employee WITH 0.85 DEGREE, for example in figure 6.12, the target class Employee Dependent should be Employee Dependent WITH 0.85 DEGREE. The dependency relationship between Employee and Employee Dependent should be fuzzy, with an 0.85 degree of possibility.

6.5 Conclusions

We present a fuzzy extended UML to cope with fuzzy as well as complex objects in the real world at a conceptual level. Different levels of fuzziness are introduced into
Abbildung 6.13: A fuzzy UML data model

the class diagram of the UML, and the corresponding graphical representations are
developed. In Figure 6.13, we give a simple fuzzy UML data model utilizing some
notations introduced in this chapter. The figure 6.14 shows a simple example for
the definition of a fuzzy class.
CLASS Young Students WITH DEGREE OF 1.0
INHERITS Students WITH DEGREE OF 1.0
ATTRIBUTES
  ID: TYPE OF string WITH DEGREE OF 1.0
  Name: TYPE OF string WITH DEGREE OF 1.0
  Age: FUZZY DOMAIN {very Young, young, old, very old}: TYPE OF integer WITH DEGREE OF 1.0
  Height: DOMAIN [60, 210]: TYPE OF real WITH DEGREE OF 1.0

Membership_Attribute name

WEIGHT
  w(ID) = 0.1
  w(Name) = 0.1
  w(Age) = 0.9
  w(Height) = 0.2

METHODS
  ...
END

Abbildung 6.14: The definition of a fuzzy class
Literaturverzeichnis

